Comparative Study of Adaptive Filter Algorithm of a QO-STBC Encoded MIMO CDMA System

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Abstract – This paper represents a comparative Study of filter algorithms Least Mean Square (LMS), Normalized Least Mean Square (NLMS) and Recursive Least Square (RLS) by considering a Quasi Orthogonal Space Time Block Code (QO-STBC) encoded Multiple Input Multiple output (MIMO) Code Division Multiple Access (CDMA) system. MIMO-CDMA system has been currently acknowledged as one of the most competitive technology. Here, the adaptive behaviors of the algorithm are studied. Implementation aspects of these algorithms are their computational complexity and Signal to Noise ratio which are also examined. Recently adaptive filtering algorithms have a nice tradeoff between the complexity and the convergence rate. In this system, by comparative study of three adaptive filter algorithms, the RLS algorithm has faster convergence rate than LMS and NLMS algorithms with better robustness to unpredictable situation and better tracking capability.

Index Terms – Adaptive filter, Least Mean Square (LMS), Normalized Least Mean Square (NLMS), Recursive Least Square (RLS), and Multiple Input Multiple output (MIMO) Code Division Multiple Access (CDMA).

1. INTRODUCTION

In today’s communication systems, for high data rate multimedia grows, several approaches have been studied to enhance the spectral efficiency [1, 2]. In future, wireless communication system design, major challenges are increased spectral efficiency and improved link reliability [3]. With the technological development, high data rate wireless communications have attracted significant interest and constitute a substantial research challenges in the context of the emerging WLANs and other multimedia networks [4, 6]. In modern digital communications, it is well known that channel equalization plays an important role in compensating channel distortion [5]. In order to meet the increasing demand for cellular mobile communication service around the globe, new digital cellular systems have been introduced during 1990, and one of the most exciting of the new technologies is CDMA [6]. International in wireless communication how to efficiently utilize the system resources, such as frequency spectrum and power, and to provide high quality transmission are the main challenges.

CDMA employs A/D converter in combination with spread spectrum technology. In a CDMA system, the same frequency can be used in every cell because channelization is done using pseudo-random codes. Data transmission involves spreading operations which are carried out by sort channelization code and long scrambling code. In this system, multi-cell interference results in degradation of BER and this characteristic affects the performance of MIMO CDMA system. MIMO antenna systems are a form of spatial diversity [7] and considered to improve the range and performance of communication system.

MIMO transmission capitalizes on fact that signals at different antennas experience independent fading in a spreading environment. Equation (1) is expression for a MIMO system with $N_t$ transmit and $N_r$ receive antennas, the received signal at the $j^{th}$ receive antenna:

$$r_j = \sum_{i=1}^{N_t} H_{ji} s_i + w_j \tag{1}$$

Where $s_i$ is symbol transmitted from $i^{th}$ transmit antenna, $H_{ji}$ is channel impulse response corresponding to propagation path between the $i^{th}$ transmit antenna and the $j^{th}$ receive antenna, and is additive (complex) Gaussian noise.

Figure 1 Block diagram of MIMO System
Combining OFDM with MIMO wireless system, have allowed for the easy transmission of symbols in the space and frequency. Different coding schemes have been developed for extract diversity from the channel e.g., Alamouti space block code (STBC) which could extract spatial and temporal diversity.

OFDM technology is a promising technique and a special case of multi-carrier transmission of signals where a single data stream is transmitted over a number of low rate subscribers. The key benefits of OFDM include high spectral efficiency, robustness to frequency selective fading, and the feasibility of low cost transceiver implementation [8]. With OFDM, we can easily mitigate the ISI because low data rates are carried by each carrier.

MIMO can be used with OFDM to improve the communication capacity and quality. When used in CDMA system, it is possible to minimize interference from neighboring cells by using different carrier permutation between two cells. For OFDM system with transmitter diversity using space time coding two or more different signals are transmitted from different antennas simultaneously [9].

Space time coding is promising technique to improve the efficiency and good performance of MIMO-OFDM systems. Space time block codes (STBC) are a generalized version of alamouti scheme. And that, they are orthogonal and can achieve full transmit diversity. STBC acts on block of data and employs special and temporal diversity techniques to mitigate the effects of scattering, fading, etc [10].

Orthogonal space time block code (O-STBC) is an important class of space time codes. To overcome the disadvantages of O-STBC, quasi-orthogonal space time block coding has been used [1, 11]. QO-STBC can achieve higher code rates and full diversity. It also provides full rate transmission and low decoding complexity. By using QO-STBC, the design of this system at low SNR can be achieved by using different type of channel equalization techniques.

In this system different adaptive filter schemes are used to compensate ISI. An adaptive equalizer is an equalization filter that automatically adapts to time varying properties of the communication channel and that self-adjust its transfer function according to an optimizing algorithm.

### 2. SYSTEM MODEL

Figure 1 shows a QO-STBC MC-CDMA system equipped with four transmit and receive antennas. At transmission time \( n \), a binary data block \( \{b[n,k]; k=0,1,\ldots\} \) is coded into different signals, \( \{t_i[n,k]; k=0,1,\ldots\} \), for \( i=1,2,\ldots,N \). So, the encoding and transmission scheme for QO-STBC system is characterized by the following coding matrix [1]:

\[
B_i(n) = \begin{bmatrix}
    b_i(4n) & -b_i'(4n+1) & -b_i''(4n+2) & b_i(4n+3) \\
    b_i(4n+1) & b_i'(4n) & -b_i''(4n+3) & -b_i(4n+2) \\
    b_i(4n+2) & -b_i'(4n+3) & b_i''(4n) & -b_i(4n+1) \\
    b_i(4n+3) & b_i'(4n+2) & -b_i''(4n+1) & b_i(4n)
\end{bmatrix} \quad (2)
\]

Where \( b_i(n) \) is the \( n \)th information symbol for user \( k \). In this system, the single data is transmitted through \( i \)th transmit antenna at the \( n \)th time slot. Let \( d_k(n) \) be the transmitted symbol and characterized by

\[
d_k(n) = \begin{bmatrix}
    d_k^{(1)}(n) \\
    d_k^{(2)}(n) \\
    d_k^{(3)}(n) \\
    d_k^{(4)}(n)
\end{bmatrix}
\]

According to the encoding scheme in (1), the coding matrix is rewritten as,

\[
B_i(n) = \begin{bmatrix}
    d_i(4n) & d_i(4n+1) & d_i(4n+2) & d_i(4n+3)
\end{bmatrix} \quad (3)
\]

From the encoding and transmission schemes described in equation (2) and (3), the coding vector \( d_k(n) \) satisfies the following relation to design efficient channel estimation and filter design for QO-STBC system.

\[
d_k(4n) = b_i(n) \\
d_k(4n+1) = M_2^T b_i(n) \\
d_k(4n+2) = M_2^T b_i' (n) \\
d_k(4n+3) = M_i b_i (n)
\]

Where \( b_i(n) = [b_i(4n), b_i'(4n+1), b_i''(4n+2), b_i(4n+3)] \) and

\[
M_2 = \begin{bmatrix}
    0 & 1 & 0 & 0 \\
    -1 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & -1 & 0
\end{bmatrix}, \quad M_i = \begin{bmatrix}
    0 & 0 & 0 & 1 \\
    0 & 0 & -1 & 0 \\
    -1 & 0 & 0 & 0 \\
    0 & -1 & 0 & 0
\end{bmatrix}
\]

After the spreading and IFFT operations, the MC-CDMA transmitted signal is given by

\[
x_k^{(i)}(n) = F^H d_k^{(i)}(n) c_k^{(i)} / n
\]

Where \( c_k^{(i)} = [c_k^{(i)}(0), \ldots, c_k^{(i)}(N-1)] \) the spreading code and \( F \) is denotes the \( N \times N \) DFT matrix. Considering the channels are frequency selective fading gains and Linear Time Invariant (LTI), Where \( h_k^{(i)}(t) = [h_k^{(i)}(0), \ldots, h_k^{(i)}(L-1)] \) is the channel impulse response from the \( i \)th transmit antenna to the \( q \)th receive antenna for the \( k \)th user and \( L \) is the maximum delay spread among all channels. Cyclic prefix is added in this system to avoid the ISI effect whose length is larger than the channel order. After discarding the corresponding cyclic prefix, the received signal at the \( q \)th receive antenna from all transmit antenna is given by
\[ r^{(q)}(n) = \sum_{i=1}^{4} x_i(n)h_i^{(q)} + \eta^{(q)}(n) \quad (6) \]

Where \( \eta = [\eta_1, \eta_2, \eta_3, \eta_4]^T \) is the uncorrelated additive white Gaussian noise \( \eta \) with variable \( \sigma^2 \). Now, perform FFT operation on the item \( X_k^{(q)}(n) \) in \( (6) \) yields

\[ FFT[X_k^{(q)}(n)h^{(q)}_k] = d_k^{(q)}(n)\Theta(F_k^{(q)}h^{(q)}_k) = d_k^{(q)}(n)c_i^{(q)}h^{(q)}_k \quad (7) \]

Where \( F_k \) contains the first \( L \) columns of \( F \) and \( c_i^{(q)} \) is an \( N \times L \) matrix, \( c_i^{(q)} = \text{diag}(c_i^{(q)}(0), \ldots, c_i^{(q)}(N-1))F_k \). Substituting \( (6) \) into \( (5) \), the frequency domain received signal at the \( q^{th} \) receive antenna is derived by

\[ y^{(q)}(n) = F_k^{(q)}(n) = \sum_{i=1}^{N} \sum_{k=1}^{L} d_k^{(q)}(n)c_i^{(q)}h^{(q)}_k + \nu^{(q)}(n) \quad (8) \]

Where \( \nu^{(q)}(n) = F_{\nu}^{(q)}(n) \). Now the signal vector \( y(n) \) is expressed as

\[ y(n) = \sum_{i=1}^{4} A_i d_i(n) + \nu(n) \quad (9) \]

Where \( A_i = [a_1^{(q)}, \ldots, a_4^{(q)}], A_i^{(q)} = (I_Q \otimes c_i^{(q)})h^{(q)}_k \).

The received QO-STBC n-codded OFDM based MIMO-CDMA block signal by stacking \( y(n) \) in four time slots, \( z(n) = [y^T(4n) y^T(4n+1) y^T(4n+2) y^T(4n+3)]^T \) is simplified as

\[ z(n) = \sum_{k=1}^{N} G_k b_k(n) + n(n) = \text{Gb}(n) + n(n) \quad (10) \]

where \( \text{G} = [G_1, \ldots, G_N], b_k = [b_1^{(q)}(n), \ldots, b_4^{(q)}(n)]^T \),

\[ G_k = [A_k^H M_2 A_k^T M_4 A_k^T M_4 A_k^H]^H \]

and

\[ n(n) = [v^T(4n) v^T(4n+1) v^T(4n+2) v^T(4n+3)]^T \]

The matrix \( G \) expresses channel impulse responses and spreading vector, and \( b(n) \) contains QO-STBC symbols for all users.

2.1 Adaptive filter

An adaptive filter may be understood as a self-modifying digital filter that adjusts its coefficients in order to minimize error function [11]. In this type of filters have self-regulation and tracking capabilities [12]. Recently adaptive filtering schemes have frequently used in signal processing, control and many other applications. These schemes become the most popular due to their simplicity and robustness.

As users gain more experience from applications and as signal processing theory matures, the adapting filtering techniques become more and more refined and sophisticated. Adaptive filtering algorithms which constitute the adjusting mechanism for the filter coefficients are in fact closely related to classical optimization techniques, and all the calculations are carried out in an off-line manner.

The basic configuration of an adaptive filter, operating in the discrete-time domain \( k \), illustrated in Figure 2. In such a scheme, the input signal is denoted by \( x(k) \), the reference signal \( d(k) \) represents the desired output signal (that usually includes some noise component), \( y(k) \) is the output of the adaptive filter, and the error signal is defined as \( e(k) = d(k) - y(k) \).

![Figure 2 Basic block diagram of adaptive filter](image)

where the error signal is used by the adaptation algorithm to update the adaptive filter coefficient vector according to some performance criterion.

2.2 Adaptive filter algorithm

The adaptive filter adapting the filter parameters varies with the application object, among these adaptive filtering algorithms [13]. Since its inception, several adaptive filter algorithms have been designed and honed. A few algorithms include LMS, NLMS and RLS algorithms are used for the comparative study of QO-STBC encoded MIMO-CDMA system [14]. These algorithms have been designed to anticipate the signal which would inevitably re-enter the transmission path and cancel it out.

2.3 Channel equalization

Channel equalization is the process of reducing amplitude, frequency and phase distortion in a channel with the intent of improving transmission performance [11]. Adaptive equalization is a technique that automatically adapts to the time varying properties of the communication channel [8]. LMS, NLMS and RLS are such popular technique that can be used for adaptive channel equalization. Every tone at each receiver antenna is associated with multiple channel parameters, which makes channel estimation difficult [9].

2.4 Least Mean Square (LMS) Adaptive Filter Algorithm

LMS algorithm has become one of the most widely used algorithms in adaptive filtering [12]. This type of algorithm
updates its weights to obtain optimal performance based on the least mean square criterion and gradient-descent methods.

\[ e(n) = d(n) - \hat{w}^T(n)x(n) \]  

(15)

Where the corresponding filter output is:

\[ d(n) = w_0^T u(n) + v(n) \]  

(16)

Where \( v(n) \) is the system noise that is independent of the input signal \( x(n) \) and \( w_0 \) is the optimal weight vector.

2.5 Normalized Least Mean (NLMS) Adaptive Filter Algorithm

In LMS algorithm, when the value of \( \mu \) is large a gradient noise amplification problem is demonstrated. For solving this problem NLMS algorithm has been used. The correction applied to weight vector \( w(n) \) with respect to squared Euclidian norm of input vector \( x(n) \) at iteration \( n \) [15]. The Normalized least mean squares filter (NLMS) is a variant of the LMS algorithm that solves this problem by normalizing with the power of the input.

\[ H(0) = \text{zeros}(p) \]  

(17)

Input signal

\[ x(n) = [x(n), x(n-1), ......., x(n-p+1)]^T \]

for \( n = 1, 2, 3, ......., p \)

The error function is \( e(n) = d(n) - y(n) \)

\[ e(n) = d(n) - h^T(n)x(n) \]  

(18)

\[ h(n+1) = h(n) + \frac{\mu e(n)x(n)}{X_n^H X(n)} \]  

(19)

If there is no interference \( (v(n) = 0) \), then the optimal learning rate for the NLMS algorithm is \( \mu_{opt} = 1 \) and is

Figure 3 System Model
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independent of the input \(x(n)\) and the real (unknown) impulse response \(h(n)\). In the general case with the interference \((v(n) \neq 0)\), the optimal learning rate is

\[
\mu_{\text{opt}} = \frac{E[|y(n) - \hat{y}(n)|^2]}{E[|e(n)|^2]} \tag{20}
\]

2.6 Recursive Least Square (RLS) Adaptive Filter Algorithm

RLS algorithm attempts to minimize the cost function. This algorithm could be recursive form to solve least squares at the moment and latest sampling value is acquired. When compared to LMS and NLMS algorithm, RLS algorithm offers a faster convergence and lower error at steady state. RLS algorithm may diverge away resulting in instability. The filter output and the error function of RLS algorithm is

\[
g(n) = \frac{\lambda^{-1} p(n-1)x(n)}{1 + \lambda^{-1} x^H(n)p(n-1)x(n)} \tag{21}
\]

where \(g(n)\) is the gain vector, \(x(n)\) is the vector of buffered input, \(p(n)\) is the inverse correlation matrix and \(\lambda^{-1}\) denotes the reciprocal of the exponential weight factor.

The output of the filter is \(y(n) = w^T(n-1)x(n)\).

The error signal is \(e(n) = d(n) - y(n)\).

The weight updated equation is

\[
w(n) = w(n-1) + g^H(n)e(n)
\]

\[
w(n) = w(n-1) + g^H(n)[d(n) - y(n)]
\]

\[
w(n) = w(n-1) + g^H(n)[d(n) - w^T(n-1)x(n)] \tag{22}
\]

Here \(g^H(n)\) is the gain coefficient. With a sequence of training data up to time, the recursive least squares algorithm estimates the weight by minimizing the following cost:

\[
\min_{w(n)} \sum_{i=1}^{m} |d(i) - u^T(i)w(n-1)|^2 + \lambda \|w(n-1)\|^2 \tag{23}
\]

Here \(x(n)\) is the input vector, \(d(n)\) is the desired response and \(\lambda\) is the regularization parameter.

3. SIMULATION RESULTS

Consider the QO-STBC OFDM based CDMA systems with four transmit antennas and four receive antennas. The transmit symbol adopts the BPSK, QPSK and DPSK modulation scheme and spreading codes of length \(N = 32\) are used. The input signal-to-noise ratio (SNR) is defined as the bit SNR at each receive antenna, which is given by.

\[
\text{SNR}(\text{dB}) = 10 \log_{10} \left( \frac{T_s P_s}{\sigma_n^2} \right) \tag{24}
\]

Where \(T_s\) indicates the number of transmit antennas. The quality of the channel estimation is measured by the root mean square error (RMSE), which is defined as

\[
\text{RMSE} = \sqrt{\frac{1}{N_m} \sum_{m=1}^{N_m} \left\| \hat{h}_m(m) - h_m(m) \right\|^2} \tag{25}
\]

where \(N_m\) is the number of Monte-Carlo-runs, \(\hat{h}_m(m)\) and \(h_m(m)\) are the estimated channel and the true channel respectively at the \(m^{th}\) runs.

In this study MATLAB-SIMULINK based QO-STBC encoded MIMO CDMA system for LMS, NLMS and RLS adaptive filter algorithms are compared on the basis of convergence speed and root mean square error (RMSE). It has been estimated filter weight, signal output and signal error in each algorithm. The SNR improvement of these algorithms, the RLS algorithm has improved SNR in dB according to it convergence speed and root mean square error (RMSE). It concludes that the best adaptive filter algorithm in this system is RLS. The proposed system is simulated in MATLAB and the simulation parameters are given in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Details</th>
</tr>
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<tr>
<td>No. of bits</td>
<td>1024</td>
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<tr>
<td>SNR</td>
<td>0-10 dB</td>
</tr>
<tr>
<td>Energy per bit</td>
<td>0.5</td>
</tr>
<tr>
<td>Modulation and Demodulation</td>
<td>DBPSK, QPSK, QAM</td>
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<td>Adaptive filter algorithm</td>
<td>LMS , NLMS, RLS</td>
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<td>Spreading Code</td>
<td>Walsh-Hadamard Code</td>
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<td>AWGN, Rayleigh</td>
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<td>States for random number generator</td>
<td>4831</td>
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<tr>
<td>FEC Code</td>
<td>Trellis Code</td>
</tr>
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Table 1 Simulation Parameters
In Figure 4 it is noticeable that the system outperforms in RLS algorithm as compared to LMS and NLMS with coherent QPSK modulation. The BER of RLS is lower than LMS and NLMS.

Figure 4 Performance of LMS, NLMS and RLS algorithm of a QO-STBC encoded MIMO CDMA with QPSK modulation

It is obvious that in Figure 5, the impact of RLS algorithm as associated to LMS and NLMS in performance enhancement of the system with coherent DBPSK modulation. The SNR of RLS is achieved better than LMS and NLMS.

Figure 5 Performance of LMS, NLMS and RLS algorithm of a QO-STBC encoded MIMO CDMA with DBPSK modulation

It is also evident that in Figure 6, the performance of RLS algorithm is achieved better than LMS and NLMS by determining BER with QAM modulation.

Figure 6 Performance of LMS, NLMS and RLS algorithm of a QO-STBC encoded MIMO CDMA with QAM modulation.

4. CONCLUSION

In this paper, we have studied the RLS, LMS, NLMS algorithms for the BER outcomes with MIMO CDMA system. In this system QO-STBC encoded MIMO CDMA embraces advantages of both MIMO system and CDMA. This system is analyzed with same number of transmit and receive antennas and different modulation schemes to compare different type of adaptive filter algorithm, simulation results show the performance improvement in the BER of the existing system by using LMS, NLMS and RLS adaptive filter algorithm.

These results show that the RLS algorithm outperforms the LMS and NLMS algorithms in terms of convergence rate and the learning behavior. These adaptive filter algorithms were analyzed and compared.

REFERENCES


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